

An Evolutionary Approach for the Split Pickup and Delivery Problem

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The split pickup and delivery problem is to find the shortest route that can provide delivery nodes with commodities collected from pickup nodes, subject to the vehicle load constraint. In particular, the split feature allows multiple visits to nodes and thus enables the vehicle to load or discharge an arbitrary portion of commodities along the route. This problem concerns the practical situations of limited capacity and insufficient commodities left for subsequent service. To resolve the split pickup and delivery problem, this study develops a memetic algorithm (MA) based on genetic algorithm and the modified 2-opt operator. The proposed MA employs a fixed-length representation carrying both the information of visiting order and the portion of requests satisfied; moreover, the modified 2-opt operator reduces the transportation cost without destroying the feasibility of route. Experimental results validate the optimization efficacy of the proposed MA in arrangement of visiting order and demand of each customer. In addition, the utility of split feature as well as its effect are examined in the empirical study.

1. Introduction

The pickup and delivery problems (PDPs) arise in various industries such as logistics and robotics. These problems consist of several nodes classified as *pickup* customers and *delivery* customers. The former supplies while the latter demands an amount of commodities. The typical objective of PDPs is to find the minimum-cost route such that the requirement of each customer can be satisfied; that is, pickup customers provide and delivery customers receive the sufficient amount of commodities. Solving the PDPs concerns vehicle routing and commodity distribution. Several variants of the PDPs have been proved to be NP-hard, and each considers particular assumptions about the transportation scenario, requirements for pickup and delivery customers, and constraints on the vehicle capacity. The PDPs generally assume equal amount of total supply and total demand and, therefore, induce an implicit constraint of visiting all customers [2]. Specifically, let $G = (V, A)$ be a complete and directed graph with vertex set $V = \{v_0, v_1, \dots, v_n\}$ in which v_0 serves as the depot and the remaining are customers. Each arc (v_i, v_j) in the arc set $A = \{(v_i, v_j) | v_i, v_j \in V, v_i \neq v_j\}$ has a non-negative cost $c(v_i, v_j)$. Let $X = \{1, \dots, e\}$ be a set of commodity entities to be transported. Each node supplies or demands a non-negative amount of commodities in accordance with commodity matrix $\mathbf{D} = [d_1 \dots d_e]$ where $|d_{ix}|$ denotes the amount of commodity $x \in X$ supplied ($d_{ix} > 0$) or demanded ($d_{ix} < 0$) by v_i . Under the assumption that the total supply and the total demand are in equilibrium for each commodity entity $x \in X$, i.e., $\sum_{v_i \in V} d_{ix} = 0$, a set of vehicles pick up or deliver commodities available or required at a vertex, forming routes such that

1. all pickup and delivery requests are served;
2. the constraint on vehicle capacity is satisfied;
3. the route cost is minimized.

Berbeglia et al. [2] presented a comprehensive survey, which classifies the PDPs into one-to-one, one-to-many-to-one, and many-to-many structures according to the types of transportation endpoints.

This study deals with the split pickup and delivery problem, which particularly allows each node to be visited more than once, considering the situation that the vehicle is incapable of loading or supplying all commodities due to insufficient vehicle capacity or shortage of commodities on board at some nodes during the pickup and delivery process. While most PDPs serve the requests by Hamiltonian cycles, the route for the split pickup and delivery problem is not necessarily a Hamiltonian cycle. The split pickup and delivery problem therefore adapts to any vehicle capacity. Notably, this study focuses on many-to-many structure, where the commodities collected from pickup customers can supply several delivery customers. The public bike sharing systems and agricultural marketing cooperative are two real-world applications of the split pickup and delivery problem, in which the key is to arrange a route for the vehicle to transport commodities such as bicycles and products to stations or retailers so as to balance the system inventory and distribute merchandise.

Multiple visits are also considered in the vehicle routing problem (VRP) where a single customer request can be served by several vehicles for flexible use of vehicle capacity. Dror and Trudeau [5] proposed the split delivery vehicle routing problem (SDVRP) and conducted several experiments to compare the SDVRP and the regular VRP. Boudia et al. [3] devised a memetic algorithm with small population size for the SDVRP. Seven local search techniques, classified into customer assignment for vehicles and commodity distribution, are used to enhance the solution quality. Enabling split delivery, however, requires more complex computation since more viable solutions should be taken into account in addition to arranging disjoint routes for vehicles. Archetti et al. [1] showed that this relaxation reduces the transportation cost when the average demand of delivery customers is within $[\frac{1}{2}Q, \frac{3}{4}Q]$, where Q denotes

the vehicle capacity. On the other hand, a large difference between customer demands and vehicle capacity makes split delivery disadvantageous. As for the PDPs, Nowak et al. [7, 8] discussed the benefit of pickup and delivery with split loads (PDPSL), which belongs to the one-to-one structure, where each pickup node has a designate delivery node.

In this study, we consider homogeneous commodity entity, namely $|X| = 1$, in the split pickup and delivery problem. The amount of commodities at node $v_i \in V$ is simply denoted by d_i . The vertex set consists of depot (or starter) v_0 with $d_0 = 0$ and two disjoint sets: $V^+ = \{v_i | v_i \in V, d_i > 0\}$ of pickup nodes and $V^- = \{v_i | v_i \in V, d_i < 0\}$ of delivery nodes. The split pickup and delivery problem seeks a minimum-cost route for a vehicle to visit each customer at least once, in order to gain commodities from pickup nodes and supply the commodities to delivery nodes. In addition, the solution is subject to the constraint that the vehicle cannot overload or attempt to serve delivery nodes with commodities more than on board.

The objective of the split pickup and delivery problem is to find a feasible permutation $\mathbf{p} = (v_0, v_{(1)}, v_{(2)}, \dots, v_{(m)})$ such that the overall cost is minimum, where $v_{(i)}$ represents the i th visiting node, m denotes the total number of visits along the route ($m \geq n$). Note that the nodes visited more than once are called ‘split nodes’. Formally, the objective is defined by

$$\arg \min \sum_{i=1}^{m-1} c(v_{(i)}, v_{(i+1)}) + c(v_0, v_{(1)}) + c(v_{(m)}, v_0) \quad (1)$$

s.t.

$$0 \leq \ell_{(i)} \leq Q, \quad \forall i \in \{1, \dots, m\} \quad (2)$$

where $\ell_{(i)}$ in constraint (2) denotes the vehicle load at $v_{(i)}$ along the visiting order, i.e., $\ell_{(i)} = \ell_{(i-1)} + \eta_{v_{(i)}}$ with $\ell_{(0)} = 0$, and $\eta_{v_{(i)}}$ represents the number of commodity units collected or discharged as visiting $v_{(i)}$. The objective (1) is to minimize the transportation cost subject to capacity limitation and non-negative vehicle load. Furthermore, for each customer $v_j \in V$, $\sum_{v_{(i)} \in \{\mathbf{p} \cap v_j\}} \eta_{v_{(i)}} = d_j$ must hold such that all pickup and delivery requests are served.

This study presents a memetic algorithm (MA) based on genetic algorithm (GA) and local search to resolve the split pickup and delivery problem. More specifically, we utilize a fixed-length representation of candidate solutions to inherently deal with the varying number of visits and the portion of requests satisfied. The variation operators are designed to change the visiting order and reassign the portion of commodities loaded or discharged in the meanwhile. The modified 2-opt operator, furthermore, serves as the local search to shorten the route without destroying its feasibility. A series of experiments are conducted to examine performance of the proposed MA and investigate the tendency of split nodes.

The remainder of this paper is organized as follows. Section 2 describes the proposed MA in detail. Section 3 presents and discusses the experimental results. Finally, we

draw conclusions and recommend the directions for future work in Section 4.

2. Evolutionary Approach

To resolve the split pickup and delivery problem, this study develops a memetic algorithm (MA) based on genetic algorithm (GA) and local search. The MA considers optimization as a search process in the solution space and implements *Darwinism* as well as *Lamarckian* or *Baldwinian* theory to imitate evolution and lifetime learning. By integrating local enhancement into canonical evolutionary algorithm, MA significantly improves its search ability and has been widely shown to enhance solution quality and convergence speed.

The proposed MA follows the reproduction process of GA and adopts local search to enhance the solution quality. Restated, the MA *represents* candidate solutions as chromosomes to enable evolutionary search in the solution space, and their respective quality (*fitness*) are evaluated through the fitness function. After initializing a set (*population*) of chromosomes, MA guides the search according to the evolutionary process inspired by nature. By mimicking natural evolution, the selection–crossover–mutation–local-search process generates new solutions and continues until reaching the predetermined termination criterion. The *selection* operator picks two chromosomes as parents from the population. The *crossover* operator then exchanges the information between these two parents to produce their *offspring* and the *mutation* operator is performed to slightly alter some genes for genetic diversity. The MA then executes local search to improve the chromosome. After the offspring population is filled, the *survival* selection implements Darwinian theory of “Survival of the Fittest” to choose the chromosomes for the next generation.

For the split pickup and delivery problem, this study uses a fixed length representation of candidate solutions to handle the varying number of split nodes. A modified 2-opt operator is adopted to improve the arrangement of visiting order. The fitness function, moreover, helps to handle the constraint on vehicle capacity and non-negative load along the route. More details about the proposed MA are described below.

2.1 Representation

A chromosome encodes a candidate solution for subsequent variation operators. An adequate chromosome representation for the split pickup and delivery problem needs to contain the information of visiting order and the portion of the commodities collected from pickup nodes and supplied to delivery nodes. This study utilizes the permutation representation in terms of demand units to fulfill the above both requirements. For a split pickup and delivery problem with a total demand amount $D = \sum_{i=1}^n |d_i|$, a permutation of D integers $\{1, 2, \dots, D\}$ represents the order of commodity transportation service, where each integer corresponds to a particular customer. Table 1 provides an example of five customers with $D = 14$ and the identification numbers to demand units. A chro-

Table 1: An example split pickup and delivery problem with five customers with $D = 14$ and the corresponding identity number (ID) for each demand unit.

Vertex	d_i	ID of demand unit
v_1	+3	1, 2, 3
v_2	+3	4, 5, 6
v_3	-5	7, 8, 9, 10, 11
v_4	+1	12
v_5	-2	13, 14

mosome is illustrated in Fig. 1, where candidate solution (1, 2, 3, 8, 7, 10, 12, 5, 6, 14, 13, 9, 4, 11) represents visiting order $(v_0, v_1, v_3, v_4, v_2, v_5, v_3, v_2, v_3, v_0)$. Accordingly, this route has two split nodes v_2 and v_3 and the multiple visits to pickup node v_2 and delivery node v_3 gradually satisfy their requests for commodities. Noteworthily, the permutation of demand units not only implies a tour for the vehicle, but inherently specifies the amount of commodities loaded or discharged through aggregating adjacent demand units belonging to the same customer.

2.2 Fitness Evaluation and Constraint Handling

The fitness function affects the selection of parents and survivors and therefore influences the search direction. An ideal fitness function must distinguish between good and bad candidate solutions. This study directly uses the objective function (1) as the fitness function $f(\mathbf{p})$ for the MA. Formally, given a chromosome \mathbf{p} representing the visiting order of nodes, its fitness value is defined as

$$f(\mathbf{p}) = \sum_{i=1}^{D-1} c(u_{(i)}, u_{(i+1)}) + c(v_0, u_{(1)}) + c(u_{(D)}, v_0), \quad (3)$$

where $c(x, y)$ gives the transportation cost between nodes x and y , and $u_{(i)}$ is the i^{th} demand unit on the visiting order and corresponds to the node requesting this pickup or delivery service.

Some chromosomes may be infeasible due to violation of the constraint on vehicle load. The fitness evaluation that can reflect the feasibility helps to render promising search directions in the constrained optimization problems. To this end, we adopt Deb's constraint handling method [4], which favors the low-cost feasible solutions and compares infeasible solutions according to the degree of constraint violation. This study defines a violation measure as follows:

$$g(\mathbf{p}) = \ell_{\text{exc}} + |\ell_{\text{neg}}| + \sum_{i=1}^D \nu_i \quad (4)$$

with

$$\ell_{\text{exc}} = \max_{i \in \{1, \dots, D\}} (\ell_{(i)}, Q) - Q \quad (5)$$

$$\ell_{\text{neg}} = \min_{i \in \{1, \dots, D\}} (\ell_{(i)}, 0) \quad (6)$$

$$\nu_i = \begin{cases} 1 & \ell_{(i-1)} < Q \text{ and } \ell_{(i)} > Q, \text{ or} \\ & \ell_{(i-1)} > 0 \text{ and } \ell_{(i)} < 0 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

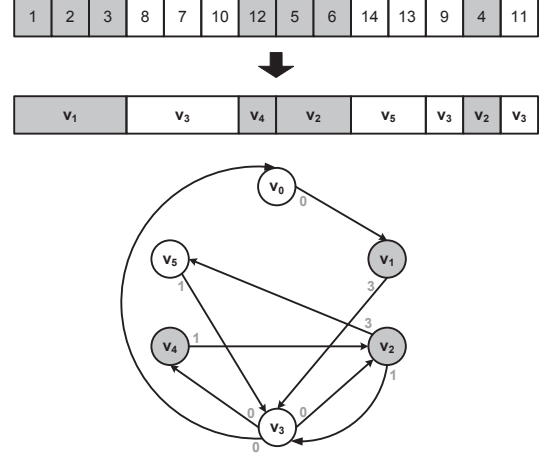


Figure 1: An example representation for the split pickup and delivery problem in Table 1. The lower string depicts the visiting order in terms of vertices according to the permutation of demand units. Pickup nodes are marked in gray; delivery nodes are marked in white. The figures upon arcs are vehicle loads.

where ℓ_{exc} represents the amount of load exceeding the vehicle capacity, ℓ_{neg} denotes the shortage of commodities on board, and ν_i indicates failing to take the advantage of the split feature along the route. The proposed violation measure $g(\mathbf{p})$ considers both the exceeding and insufficient amounts of vehicle load; therefore, the incapability of properly splitting the demand at a particular node will be penalized. As the basis of parent selection and survivor selection, this fitness evaluation handles the constraint by leading the search toward feasible solutions.

2.3 Genetic Operators and Local Search

The genetic operators such as selection, crossover, and mutation facilitate exploring the search space. This study adopts the binary tournament selection as parent selection operator. Based on the proposed fitness evaluation, the fitter of two chromosomes randomly picked from the population is selected as a parent. The crossover operation exchanges and recombines the genetic information of parents, and the mutation operation slightly changes the composition of offspring to introduce diversity to the population. In view of the permutation representation for the split pickup and delivery problem, we employ the order crossover and inversion mutation [6]. The order crossover randomly chooses two cut points to divide each parent (route) into two segments. An offspring is generated by directly inheriting a segment from one parent and filling the remainder genes with absent demand units according to the visiting order of the other parent. The inversion mutation operator reverses the order of genes in a randomly determined segment. The order crossover and inversion mutation can avoid duplicate appearances of single identity in an individual and thus satisfies the requirement of a legal permutation. In addition to the change of visiting order, the variation operators implicitly alter the portion of demands satisfied.

Table 2: Parameter setting for MA.

Parameter	Value
Representation	Permutation of demand units
Population size	500
Initialization	Random
Parent selection	Binary tournament
Crossover	Order crossover ($p_c = 1.0$)
Mutation	Inversion ($p_m = 1.0$)
Local search	Modified 2-opt
Survival selection	$(\mu + \lambda)$
Termination	15000 generations

Moreover, we employ the modified 2-opt operator [9] to implement the local enhancement in MA. The modified 2-opt iteratively inverts the segments within permitted substrings in the chromosome to achieve a shorter route without breaking the feasibility in vehicle load. The substrings are determined by separating the chromosome on transitions from pickup node to delivery node and vice versa, given that these changeover points coincide with critical loads along the route. Restated, the required capacity takes place at one of the transitions from pickup node to delivery node and the minimal vehicle load exists in the opposite situations. Note that the subtotal of demands keeps fixed due to the transitive feature disregarding the order within a substring, and the critical loads are therefore held, which maintains the solution feasibility. After generating a population of offspring, only the fittest individuals of the union of all parents and offspring are selected to survive into the next generation.

3. Experimental Results

This study conducts a series of experiments to evaluate the effectiveness of the proposed MA on the split pickup and delivery problem. The benchmark instances are modified from the problem instances used in [9]. In modifying the instances for the split pickup and delivery problem, we limit demand range in $[-5, +5]$. The test suit is denoted by X/Y, where X is the original instance name and Y represents the number of nodes in the split pickup and delivery problem instance. For example, n20mosA/17 denotes a 17-node instance of split pickup and delivery problem modified from n20mosH. The cost between two nodes is defined as their distance. The split pickup and delivery problem is to find the shortest route that can provide delivery nodes with commodities collected from pickup nodes, where the split feature enables multiple visits to a node. In other words, the vehicle can load or discharge any portion of commodities supplied by pickup nodes or demanded by delivery nodes, respectively. Table 2 summarizes the parameter setting for the MA used in the experiments. In particular, the termination criterion for n100mosA/78 and n100mosB/89 is extended to be 40000 generations due to their large problem scale. Each test instance includes 30 independent runs of MA.

First, we investigate the average route lengths and the influence of vehicle capacity upon them. According to Table 3, the average route cost decreases as vehicle capacity Q increases in that a small capacity raises the requirement for splitting nodes and thus increases the route cost. Figure 2 illustrates the variation of route cost against vehicle capacity, where a significant increase in route cost occurs at $Q = 3, 5$. These results also validate the optimization efficacy of the proposed MA in arrangement of visiting order and demand of each customer. In addition, the number of split nodes in the best solutions demonstrates the tendency to aggregate visits: For $Q = 10, 20$, the average number of visits is lower than 2.0 except for n100mosA/78, implying that the vehicle visits most nodes only once. Due to multiple visits, the solution space can reach five-fold of the customer number in the worst case; for instance, the chromosomes for n100mosA/78 have 390 genes. These features make the split pickup and delivery problem a difficult constrained optimization problem.

Figure 3 presents the proportions for each number of visits. The results reveal the tendency to visit the majority of customers exactly once even with the lowest vehicle capacity. The proposed MA inclines to aggregate the service of demand units for large vehicle capacity. Additionally, more visits are required on the instances with more customers as shown in Fig. 3 and Table 3. This outcome reflects the complication of search for a feasible minimal-cost route that includes multiple visits to certain nodes and considers their orders.

Furthermore, Fig. 4 shows the anytime behavior of the proposed MA. For consistency of comparison, the y coordinate denotes the ratio of route cost to the known best cost. The results demonstrate that, in general, a large capacity leads to fast convergence since this loose constraint involves more feasible solutions and thus enables a relatively smooth way to reach the optimum. The results show a drastic improvement in convergence speed for $Q = 10, 20$ on all test instances. The long generations of infeasible solutions for n100mosA/78 and n100mosB/89 with $Q = 3$ reconfirm the intensification on splitting visits and on the constraint.

4. Conclusions

The split pickup and delivery problem aims to find the shortest route that can provide delivery nodes with commodities collected from pickup nodes subject to non-negative vehicle load and capacity limitation. An important feature of this problem is its allowance for multiple visits to a single node, which enables the vehicle to load or discharge arbitrary portion of commodities supplied by pickup nodes or demanded by delivery nodes, respectively. The need for split nodes arises especially when the vehicle is incapable of loading or supplying all commodities due to insufficient vehicle capacity or shortage of commodities on board at some nodes along the route. The split pickup and delivery problem therefore adapts to any vehicle capacity.

This study designs an MA based on genetic algorithm and 2-opt local search for the split pickup and delivery prob-

Table 3: Average route cost (cost), average number of split nodes (#splits) and average number of visits per node (#visits) in the best route over 30 trials on 10 test instances with different vehicle capacity Q . Note that infeasible solutions are disregarded in the averages.

instance	$Q = 3$			$Q = 5$		
	cost	#splits	#visits	cost	#splits	#visits
n20mosA/17	5414.00	2.17	2.00	4552.77	0.60	1.53
n20mosB/15	4872.23	2.00	2.00	4021.77	0.87	1.87
n30mosA/28	8706.03	6.87	2.00	6385.20	1.37	1.67
n30mosB/25	6951.17	5.83	2.00	5669.90	1.53	1.90
n40mosA/33	8041.60	8.80	2.01	6530.17	2.33	1.98
n40mosB/34	7877.53	9.57	2.03	6002.67	2.43	2.00
n50mosA/41	8579.23	9.23	2.01	7056.40	2.53	2.01
n50mosB/43	10633.17	15.23	2.00	8191.77	4.77	2.00
n100mosA/78	18897.43	27.64	2.03	13241.50	9.03	2.06
n100mosB/89	17072.50	41.17	2.17	11243.70	7.83	2.01

instance	$Q = 10$			$Q = 20$		
	cost	#splits	#visits	cost	#splits	#visits
n20mosA/17	3822.43	0.23	1.23	3702.73	0.03	1.03
n20mosB/15	3825.10	0.03	1.03	4003.33	0.00	1.00
n30mosA/28	5168.03	0.33	1.30	4977.60	0.27	1.23
n30mosB/25	4962.57	0.77	1.57	4812.87	0.80	1.47
n40mosA/33	5597.23	1.03	1.57	5173.20	0.47	1.23
n40mosB/34	5465.00	0.93	1.68	5117.93	0.33	1.31
n50mosA/41	6283.13	0.53	1.30	6341.40	0.70	1.37
n50mosB/43	6333.97	0.57	1.50	5972.20	0.33	1.17
n100mosA/78	10204.30	4.47	2.00	8887.67	3.43	1.90
n100mosB/89	8912.43	1.90	1.77	8263.47	1.57	1.67

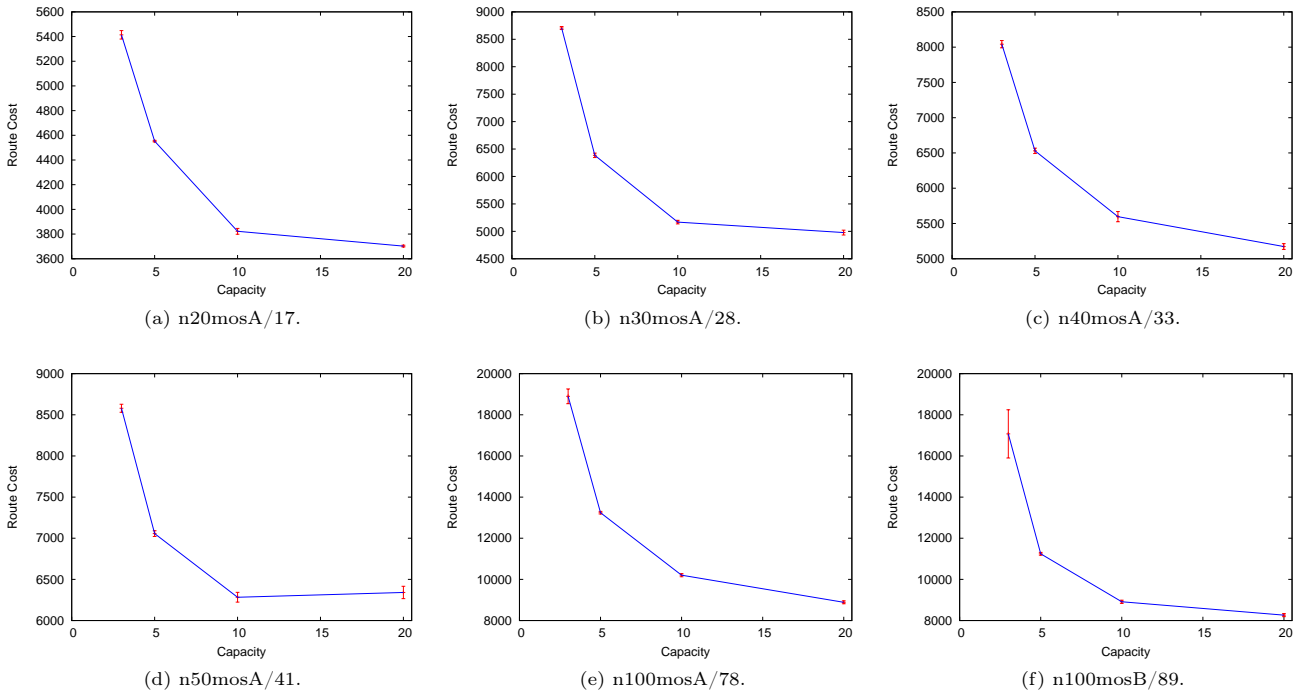


Figure 2: Average route cost against vehicle capacity Q . The red lines indicate the standard errors.

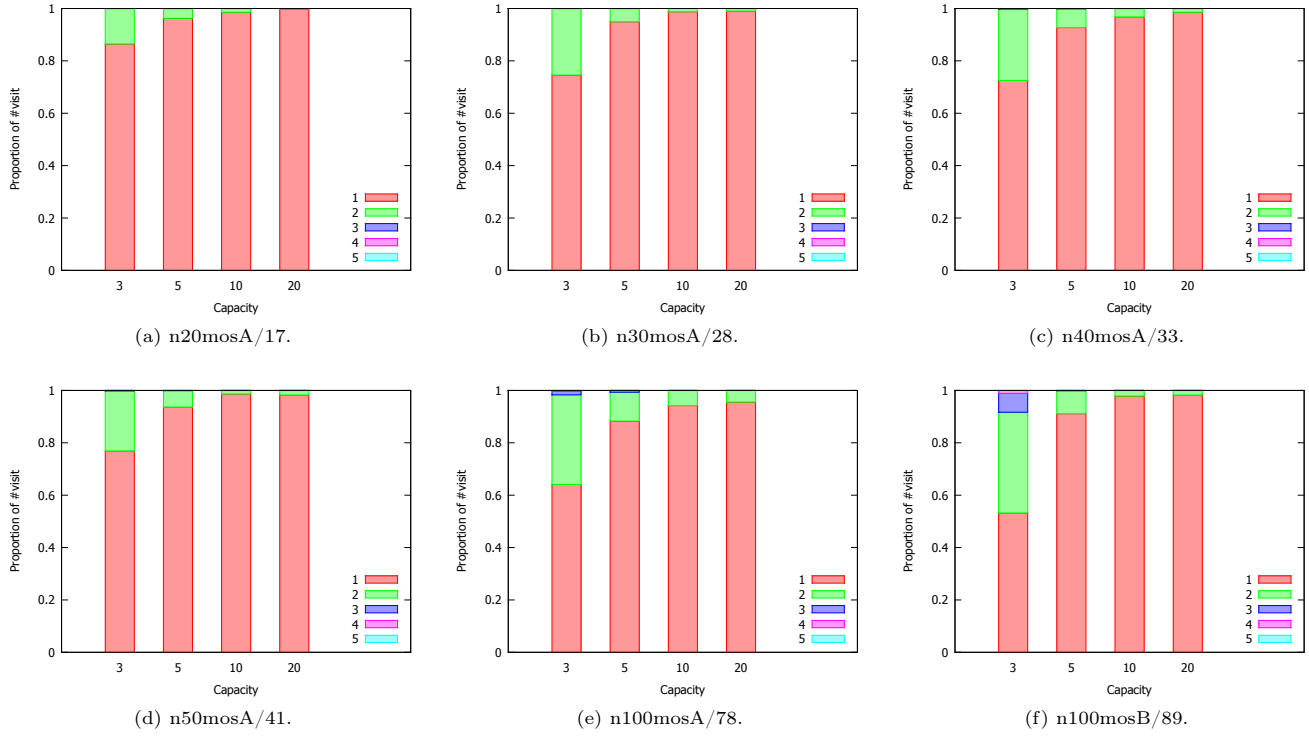


Figure 3: Proportion of the number of visits.

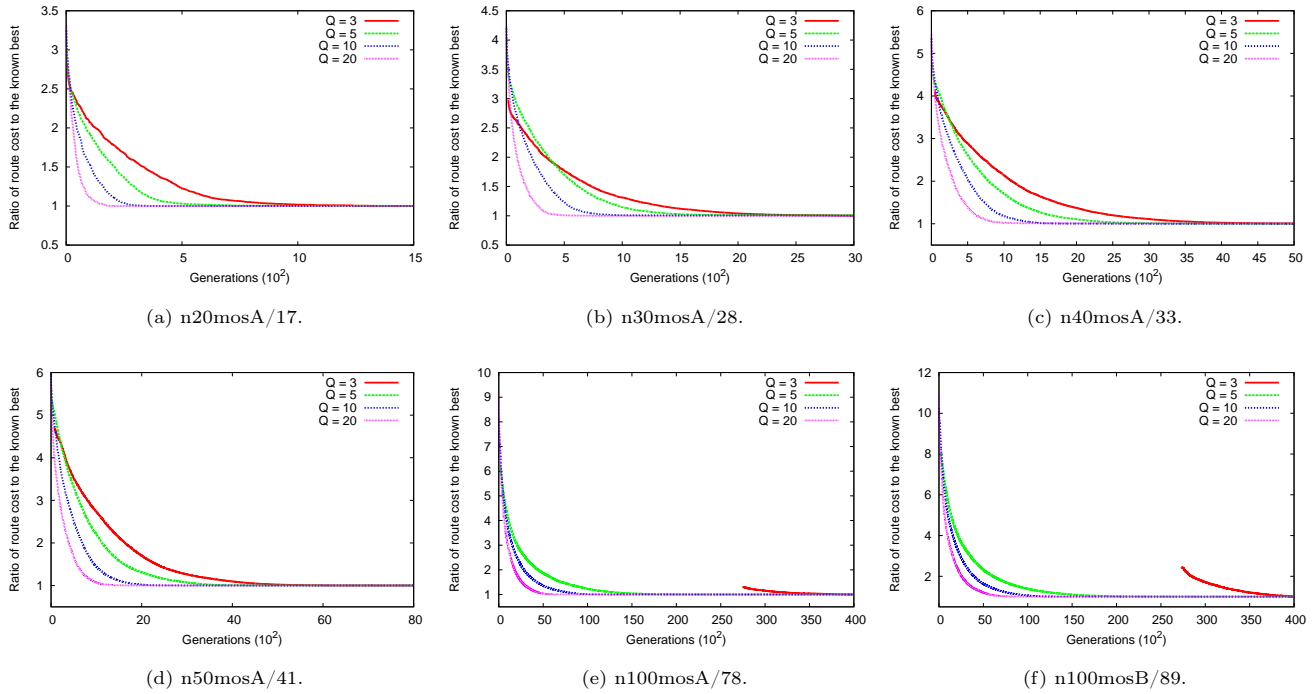


Figure 4: Anytime behavior of the proposed MA.

lem. A fixed-length chromosome representation is proposed to indicate the visiting order of customer nodes as well as the amount of commodities picked up or delivered at each node. The variation operators are devised to simultaneously change the genetic information of candidate solutions. The modified 2-opt operator, furthermore, serves as the local search to shorten the route without destroying its feasibility.

A series of experiments are carried out to examine performance of the proposed MA and characterize the split pickup and delivery problem. The experimental results validate the optimization efficacy of the proposed MA in arrangement of visiting order and demands of customers. In addition, the results show that small vehicle capacity raises the requirement for splitting nodes and thus increases the route cost; nevertheless, the vehicle tends to visit the majority of customers exactly once even with small capacity.

Future work includes modification of algorithms for the related problems. Extension to design of heuristic operators and performance comparison is also an important direction for the study on the split pickup and delivery problem.

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